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AN ESTIMATE OF THE EFFECT OF MER
(MULTIPLE EJECTION RACKS) STRUCTURAL
DYNAMICS ON STORE SEPARATION

Lercy Devan

Naval Weapons Laboratory
Dahlgren, Virginia

February 1974

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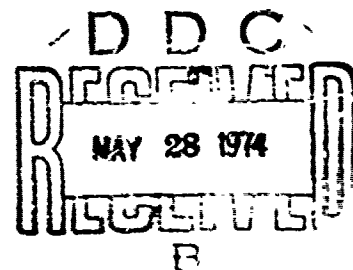
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by

Leroy Devan
Warfare Analysis Department



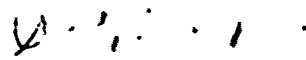
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FOREWORD

This report describes work directed toward estimating the effect of multiple Ejection Rack (MER) flexibility on store ejection conditions. This work was sponsored by the Naval Air Systems Command under AIRTASK A3200000/009B/3F232000. The report was reviewed by Dr. Thomas A. Clare, Head, Flight Dynamics Group, and Russell D. Cuddy, Head, Aeroballistics Division.

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model is based upon "Bernoulli" pitch and yaw deflections and torsional rotation of the PIER beam; wing, pylon, hanger, shoulder pad, and ejector unit deformations are neglected.

Detailed development for the present report is confined to the pitch plane bending and store pitch dynamics case. For small pitch angles a linear set of kinematic and dynamic equations for the store ejection phase is obtained. Sample computations are given for two stores and constant aircraft pull-up rates. A 4-g pull-up produces the greatest structural deformations. However, the pull-up maneuver alone affects the ejection pitch rate and velocity to a greater extent than the structural deformation. A break even point occurs for smaller pull-up rates. For the pitch bending case, flexibility probably accounts for no more than a 10% deviation from the rigid case. Flexibility affects the ejection angular rate more significantly.

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ABSTRACT

Structural flexibility has long been suspected as being a factor affecting the ejection phase dynamic response of stores. After hook opening, in-carriage loads, associated with a released store, and an ejection recoil force deflect the MER beam away from its initial position. This in turn leads to a lower ejection velocity, lower ejection pitch rate, and an induced rolling moment. A theoretical structural dynamic model is based upon "Bernoulli" pitch and yaw deflections and torsional rotation of the MER beam; wing, pylon, hanger, shoulder pad, and ejector unit deformations are neglected.

Detailed development for the present report is confined to the pitch plane bending and store pitch dynamics case. For small pitch angles a linear set of kinematic and dynamic equations for the store ejection phase is obtained. Sample computations are given for two stores and constant aircraft pull-up rates. A 4-g pull-up produces the greatest structural deformations. However, the pull-up maneuver alone affects the ejection pitch rate and velocity to a greater extent than the structural deformation. A break even point occurs for smaller pull-up rates. For the pitch bending case, flexibility probably accounts for no more than a 10% deviation from the rigid case. Flexibility affects the ejection angular rate more significantly.

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I. INTRODUCTION

Structural flexibility has long been suspected as being a factor affecting the ejection phase dynamic response of stores released from Multiple Ejection Racks (MER). Reference 1, which provides a good summary of the problem areas of store separation, contains several qualitative references to flexible MER effects. Reference 2 considers the effect of rack deformation on ejection velocity from a simple energy consideration for a uniform cantilever beam with a half sine pulse ejector force. Reference 3, considers a simple one degree of freedom model for a bomb rack and separating store with step function ejector force.

The objective of the present report is to approximate the structural dynamic response of the MER ejector beam and the ejection phase of store separation in more detail. Detailed development will be limited to an aircraft pulling up at a constant rate, pitch plane bending, and pitch plane motion of the released store.

II. STRUCTURAL DYNAMIC MODEL OF A MER EJECTOR BEAM

Figure 1 is a schematic of a MER 7 or 9 ejector beam. The following sequence of events describes most of the pertinent ejection phase phenomena, associated with a flexible rack, which occur after hook opening:

- (1) The in-carriage weight, acceleration, and aerodynamic loads, associated with a released store, are removed from an initially deflected MER beam.
- (2) The ejection force displaces the store until the ejector foot is extended about 3.25 inches. Simultaneously a recoil force acts on the ejector unit.
- (3) The MER beam deflects away from its initial position. This changes the position of the ejector foot relative to the store. For a single store or long time-interval string release flexibility contributes to a lower ejection velocity, lower ejection pitch rate and an induced rolling moment. For a ripple or salvo mode the beam is initially in motion and release phenomena are very much dependent on the release interval.

In Figure 1 are shown coordinate systems used. x, y, z is a right-handed coordinate system embedded in the rigid MER beam. x coincides approximately with a neutral axis, x and z are parallel to the aircraft symmetry plane. X, Y, Z is parallel to x, y, z and embedded in the rigid aircraft frame with origin at the initial rigid, e.g. location of the store being released. x', y', z' is a right-handed set of store body coordinates.

Major assumptions and features of the model are enumerated below: they render the problem tractable and, hopefully, physically representative.

- (1) Wing, pylon, hanger, shoulder pad, and ejector unit deformations are neglected.
- (2) Coriolis and centripetal accelerations induced by deformation are neglected.
- (3) "Bernoulli" beam pitch and yaw deflections are allowed for the MER beam. Torsional deformation is assumed to induce no longitudinal stresses. Longitudinal oscillations are neglected.
- (4) Lug connections from the parent rack to the MER are assumed to be pinned. Parent rack sway brace reactions are assumed to be spring forces. It is assumed that sway braces do not break contact with the MER during deformation (questionable for the AERO-7A rack).

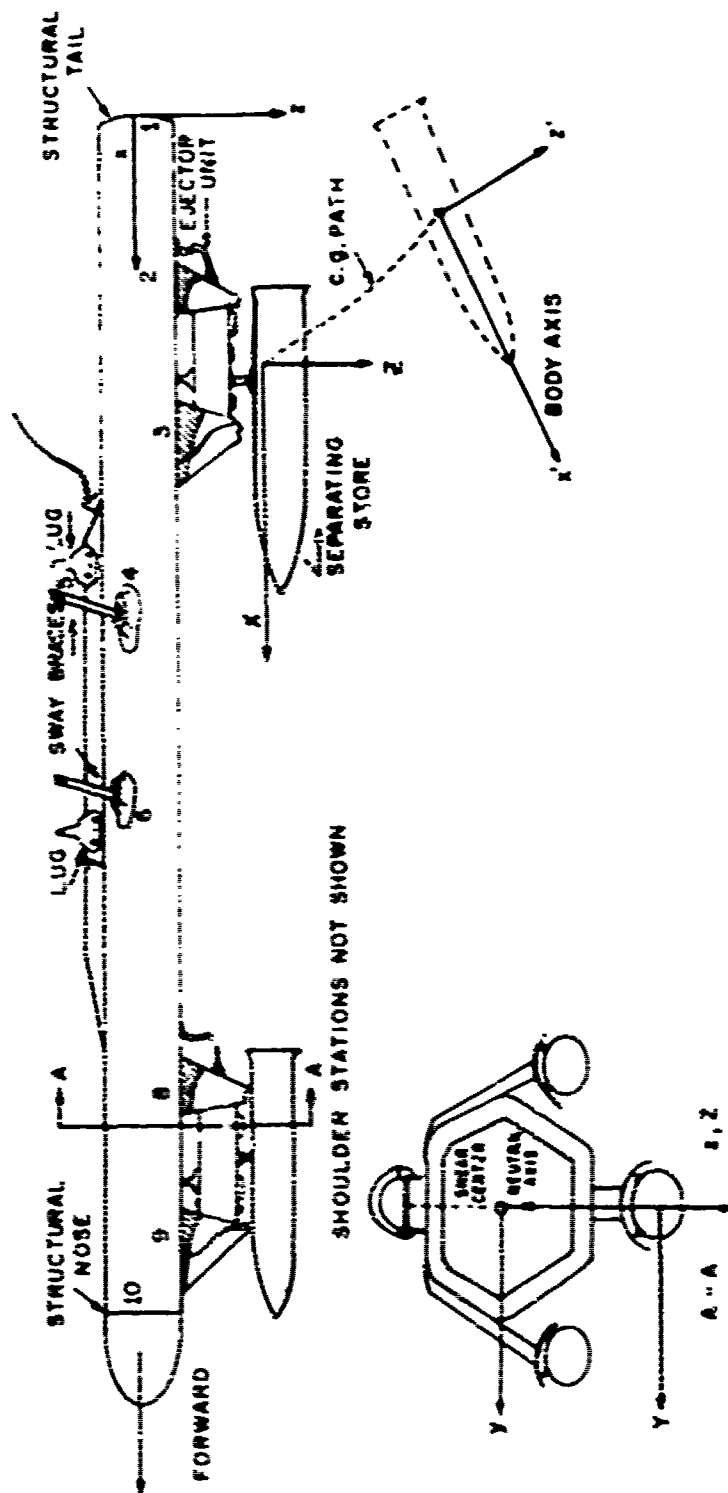


FIGURE 1

Ejection From a Flexible Mer Ejector Beam

- (5) Static and dynamic aeroelastic forces on the MER are neglected due to lack of experimental data.
- (6) Kinematics of the ejecting foot and in-carriage store c.g. are computed from beam deflections and rotations at beam-ejector unit connection cross sections.
- (7) For a given cartridge, a single ejection force-time function is assumed for all store loads. Ejector force dynamics are assumed to be independent of beam motion.
- (8) Beam motion acceleration, aircraft acceleration, ejection force, and aerodynamic loads for in-carriage stores are translated into beam loads by statically determinant computations.
- (9) For the applications given all stores in-carriage are identical.

A short stick release of stores will not deform the pylons and wings greatly. For a larger number of stores released this assumption must be reexamined. Some of the assumptions involving structural deformations, mass distributions, and kinematics could be improved if supporting experimental data were available.

A. Influence Coefficients

The MER beam weighs about 210 pounds. More than half of the beam weight is concentrated between the ejector unit support cross sections. Flexibility is more important for heavier stores. Hence, the ratio of effective beam to store mass is expected to be 1/5 or less and therefore, the beam mass distribution need not be specified with great accuracy. Therefore, only deflections at a relatively few cross sections such as those enumerated 1-10 in Figure 1 need be considered.

For the AERO-7A parent rack at the sway brace cross sections

$$F_z = -k\xi \quad (1)$$

For unit forces placed at a collocation point (not including lugs where deflections are zero) deflections are computed at all collocation points (force flexibility coefficients). For unit couples, placed at ejector unit and lug connection cross sections, deflections are computed at the collocation points (moment flexibility

coefficients). Additional assumptions are neglect of shear deflection and a constant value of second moment of area between collocation points or between collocation points and lugs. Couples are due to horizontal forces and contribute less to deflections than vertical forces.

A general equilibrium equation may be written as

$$\zeta(x_1, t) = \int_0^L C_f(x_1, \xi) p_z(\xi, t) d\xi + \sum_{k=1,6} C_{Mik} M_k \quad (2)$$

p_z and M_k include inertial, gravity, maneuver and aerodynamic forces.

B. Natural Frequencies and Normal Modes

p_z is now decomposed into distributed and concentrated forces. The integral for distributed forces is replaced by a numerical approximation (trapezoidal rule is adequate).

$$\zeta_i = \sum_{j=1,6} C_{fij} \left(-m_j \frac{d^2 \zeta_j}{dt^2} + \bar{F}_{zj} \right) + \sum_{k=1,6} C_{Mik} M_k \quad (3)$$

The set of equations (3) are known as collocation equations (see for instance Reference 4). i or j indices refer to collocation points. The aft ejector unit support points are at $i = j = n_1, n_2$ or $k = 1, 2$. Lug connection points are at $k = 3, 4$. The forward ejector unit support points are at $i = j = n_5, n_6$ or $k = 5, 6$. m_j are effective concentrated masses at the collocation points. \bar{F}_{zj} are net vertical forces at a cross section. M_k are net couples and are due to horizontal forces.

It is convenient to choose an unloaded horizontal beam subject to only its own weight as a reference condition. \bar{F}_{zj} may be further broken down into concentrated and distributed contributions

$$\bar{F}_{zj} = m_j [g(1 - \cos \theta_0) + \dot{\theta} V_0] + F_{zj} \quad (4)$$

F_{zj} is zero except at the ejector unit support points. The concentrated forces at the support points are due to loads on the individual stores.

In Figure 2(a) is shown the force system assumed to act on a centerline store and hanger supports removed as a free body from the MER beam. The hangers and ejector unit are not shown, only the forces and points of action

The assumption of a pinned connection with equal horizontal forces at the hanger-beam connection point renders the computation of f_{xc} , f_{zcf} , and f_{zca} statically determinant. From moment and force equilibrium

$$\begin{aligned} f_{xc} &= F_{xc}/2 \\ f_{zca} &= (F_{zc}x_f + M_{yc} + F_{xc}z_c)/x_h \\ f_{zcf} &= (F_{zc}x_a - M_{yc} - F_{xc}z_c)/x_h \end{aligned} \quad (5)$$

In Figure 2(b) is shown a sideview of the force system acting on a shoulder station store. Again from equilibrium

$$\begin{aligned} f_{xs} &= F_{xs}/4 \\ f_{zsa} &= [F_{zs}x_f + F_{xs}(z_s - \Delta z/2) + M_{ys}]/x_h \\ f_{zsf} &= [F_{zs}x_a - F_{xs}(z_s - \Delta z/2) - M_{ys}]/x_h \end{aligned} \quad (6)$$

Support point net forces and moments are given by equations like

$$\begin{aligned} F_{za} &= f_{zca} + 2f_{zsa} \\ M_a &= M_f = z_1 f_{xc} + (2z_2 + \Delta z)f_{xs} \end{aligned} \quad (7)$$

After ejection is initiated the centerline store forces of a separating store are replaced by the ejection force and (5) is replaced by

$$\begin{aligned} f_{xc} &= 0 \\ f_{zca} &= -F_e(t)(x_f - x_c)/x_h \\ f_{zcf} &= -F_e(t)(x_a + x_c)/x_h \end{aligned} \quad (8)$$

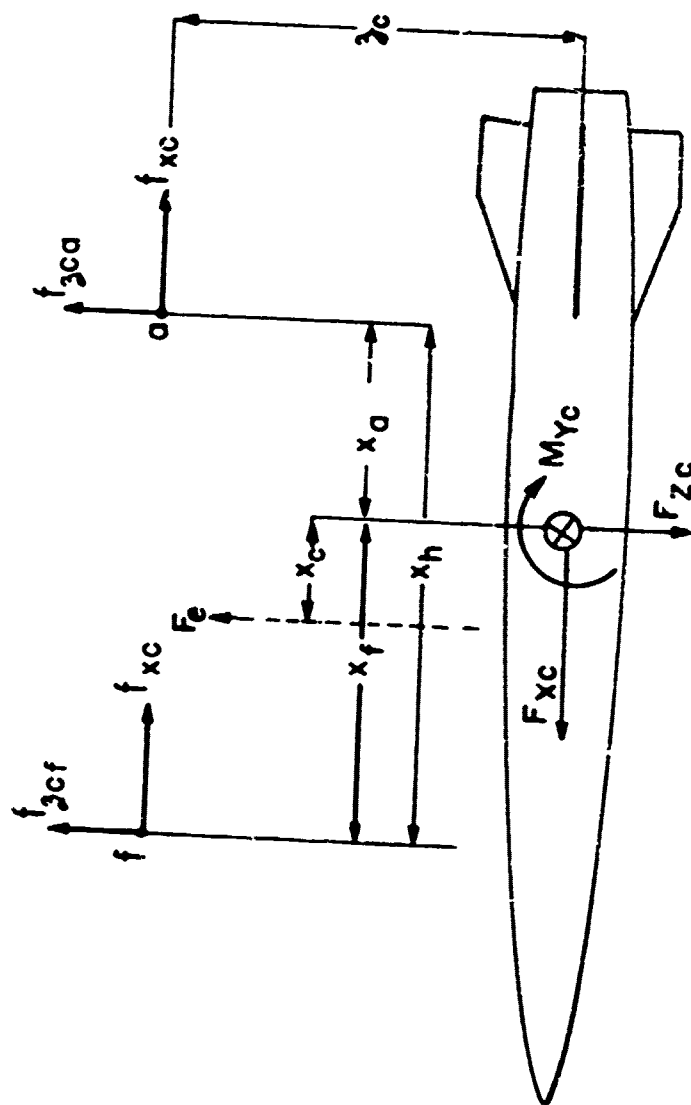


FIGURE 2(a)
Force System on Centerline Store and
Ejector Unit Supports

FIGURE 2(b)
Force System on Shoulder Store and Ejector Unit Supports

The couples at the lugs are given by

$$M_{L_1} = M_{L_2} = \frac{1}{2} z_c [F_{X1} + 2(F_{X3} + F_{X4}) + F_{X2}] \quad (9)$$

Thus far the sets of equations (6)-(9) are valid for all store loads. Some of the stores and hence loads may be absent.

Inertial loads at the store, e.g. must be defined in order to complete formulation of the characteristic equations.

It is assumed that the store and ejector unit assume the angular attitude of the secant of the deflection curve between support cross sections. For a centerline store

$$\begin{aligned} F_{X_c} &= - \frac{M_s z_c (\ddot{\xi}_s - \ddot{\xi}_f)}{(x_s + x_f)} \\ F_{Z_c} &= \frac{- M_s (\lambda_s \ddot{\xi}_f + \lambda_f \ddot{\xi}_s)}{(x_s + x_f)} \\ M_{Y_c} &= \frac{- B(\ddot{\xi}_s - \ddot{\xi}_f)}{(x_s + x_f)} \end{aligned} \quad (10)$$

Shoulder station equations are similar with z_c replaced by z_s . The horizontal acceleration contributes least to the natural frequency.

Substitution of the sets of equations (6)-(10) into equation (5) with only inertial forces acting yields a homogeneous set of equations of the form

$$\xi_i = - \sum_{j=1, \dots, N} K_{ij} m_j \frac{d^2 \xi_j}{dt^2} \quad (11)$$

The eigenvalue problem is given by

$$\lambda Z_i = \sum_j K_{ij} m_j Z_j \quad (12)$$

K_{ij} is not symmetric and hence the formulation is non self-adjoint. The characteristic equation coefficients are obtained by application of the Cayley-Hamilton theorem (Reference 5). A good first iteration for the dominant eigenvalue is given by Sylvester's theorem (Reference 5). The eigenvector for the eigenvalue λ_n is given by

$$\lambda_n Z_{in} = \sum_j K_{ij} m_j Z_{jn} \quad (13)$$

The adjoint eigenvector, \dot{Z}_{in} for λ_n is given by

$$\lambda_n \dot{Z}_{in} = \sum_j K_{ji} m_j \dot{Z}_{jn} \quad (14)$$

Orthogonality is given by

$$\sum_j m_j Z_{jn} \dot{Z}_{jm} = 0 \quad (15)$$

C. Dynamic Response of MER Beam to Store Release

The reference condition from which the deflection is computed is the horizontal MER beam under its own weight. Normally only in-carriage airloads are available (airborne or wind tunnel) for the store about to be released. However, in-carriage airloads for all stores for a given release condition were available for the A-7D from LTV for the M-117 bomb (Reference 6). If only airloads are available for the store about to be released, the reference condition must be the deflected position just prior to release.

Incremental loads due to static aeroelastic deformation are not considered. However, the deformation is considered when the initial angle of attack of the released store with the free stream is computed for the ejection phase. It is assumed that during the ejection phase the airloads on stores remaining in-carriage do not change. Static measurements of in-carriage airloads indicate fairly large differences between a six-bomb and five-bomb configuration or more generally between an N-bomb and N-1-bomb configuration. Conceivably, transient in-carriage loads could be measured by in-carriage balances. These dynamic loads would be due to MER beam vibrations and released store transition through the interference near-flow field.

Without any experimental data available the transient between static conditions would have to be assumed. Such considerations have not been applied here.

The general equations of motion may be written as

$$\ddot{\xi}_i(t) = - \sum_{j=1, \dots, N} K_{ij} m_j \frac{d^2 \xi_j}{dt^2} - K_{ci} F_c(t) + L_i \quad (16)$$

L_i is the deformation due to constant air, gravity, and maneuver loads.

An eigenvector expansion is assumed (truncated at N modes)

$$\xi_i = \sum_{n=1, \dots, N} T_n(t) Z_{in} \quad (17)$$

Substitution of (17) into (16) with application of (15) yields

$$\ddot{T}_n + \omega_n^2 T_n = \frac{\omega_n^2}{M_n} \left[\sum_{i=1, \dots, N} m_i \dot{Z}_{in} L_i - F_c(t) \sum_{i=1, \dots, N} m_i \dot{Z}_{in} K_{ci} \right] \quad (18)$$

(18) is readily solved as

$$\begin{aligned} T_n = T_n(0) \cos \omega_n t + \frac{\dot{T}_n(0)}{\omega_n} \sin \omega_n t + \sum_{i=1, \dots, N} \frac{m_i \dot{Z}_{in} L_i (1 - \cos \omega_n t)}{M_n} \\ + \frac{\omega_n}{M_n} \sum_{i=1, \dots, N} m_i K_{ci} \dot{Z}_{in} \int_0^t \sin \omega_n (t - \tau) F_c(\tau) d\tau \end{aligned} \quad (19)$$

$$T_n(0) = \sum_{i=1, \dots, N} \frac{\xi_i(0) m_i \dot{Z}_{in}}{M_n}$$

$$\dot{T}_n(0) = \sum_{i=1, \dots, N} \frac{\dot{\xi}_i(0) m_i \dot{Z}_{in}}{M_n} \quad (20)$$

Equations (19) and (17) give the beam response to store release and ejection force. $\xi_1(0)$ is non zero only for the ripple case. The response is computed for about 100 milliseconds and stored for later combination with store dynamics during the ejection phase.

III. EJECTION PHASE STORE DYNAMICS

A. Ejection Kinematics

Kinematics of the store and ejector unit are computed relative to the X, Y, Z coordinate system. Necessary geometry is shown in Figure 3.

f and a are the same points as shown in Figure 2. ϕ and θ_e are small angles. p is a point on the bottom surface of the ejector unit where the piston foot exits. p_0 is the rigid case location of point p. pp' remains perpendicular to the bottom surface of the ejector unit.

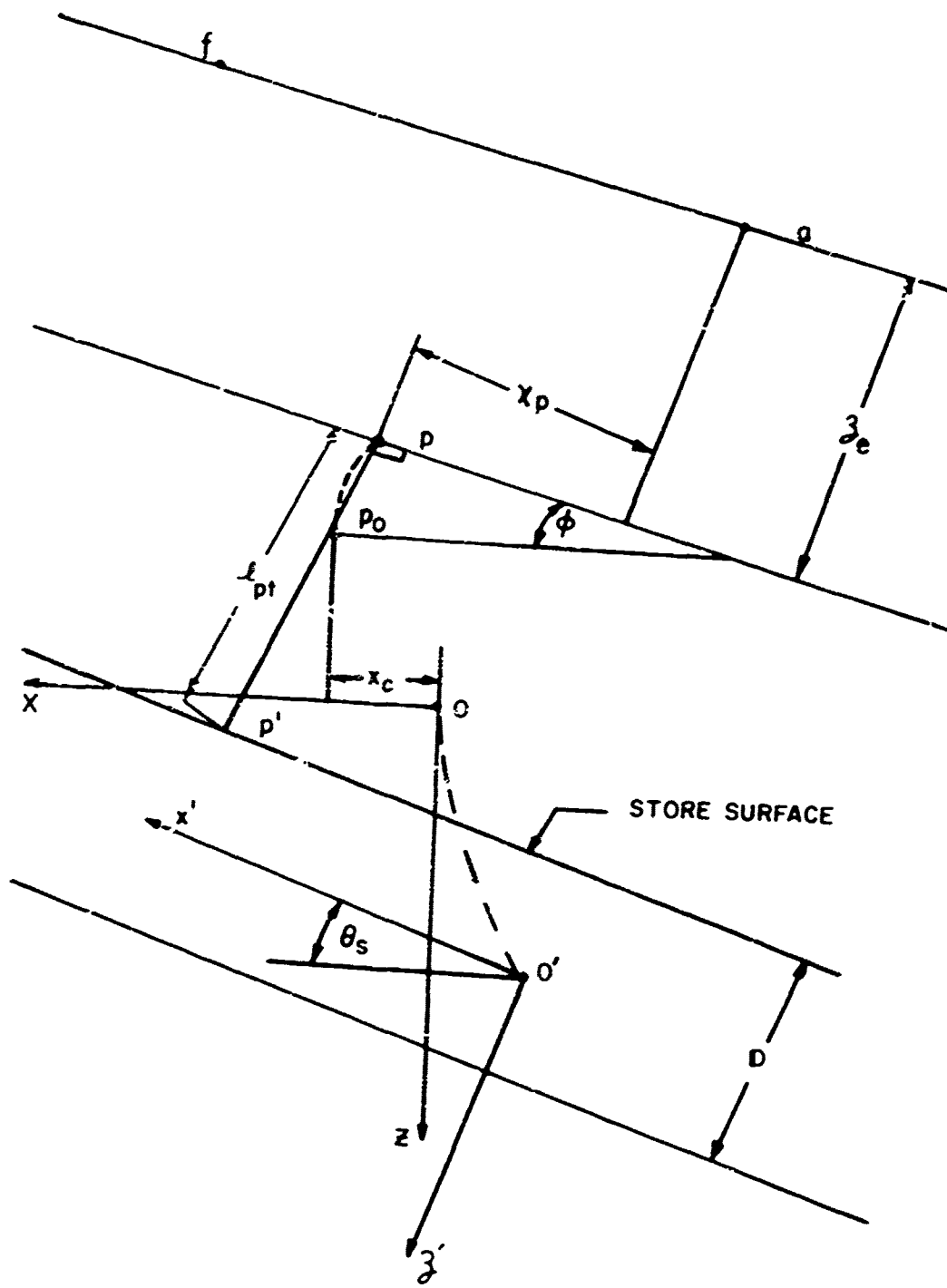
The point p coordinates are given as

$$\begin{aligned} X_p &\approx x_e + z_e \frac{\xi_a - \xi_f}{\lambda_h} = x_e + z_e \phi \\ Z_p &\approx \xi_a + z_e - x_p \phi - l_e \end{aligned} \quad (21)$$

Initial conditions for the store are given by

$$\begin{aligned} \theta_e(0) &= \phi(0) \\ X_e(0) &= z_e \phi(0) \\ Z_e(0) &= \xi_a(0) - (x_p - x_e) \phi(0) \\ \dot{\theta}_e(0) &= \dot{\phi}(0) \\ U_e(0) &= z_e \dot{\phi}(0) \\ W_e(0) &= \dot{\xi}_a(0) - (x_p - x_e) \dot{\phi}(0) \end{aligned} \quad (22)$$

Subsequent values of X_e and Z_e are governed by the dynamics of store separation.



Ejection Phase Geometry

The length of the piston throw (pp') is given by

$$l_{pp'} \approx Z_s \cdot \frac{D}{2} - Z_p + \theta_s (X_s - X_p) \quad (23)$$

The force moment arm is given by

$$l_m \approx X_p - X_s + \theta (Z_s - Z_p) \quad (24)$$

B. Ejection Equations of Motion in X, Z Coordinates

In Figure 4 is shown some additional velocity vector and geometry information.

The equations of motion in body coordinates are

$$\begin{aligned} M_s (\dot{u}' + q'w') &\approx \frac{1}{2} \rho V'^2 S C_A - M_s g \sin \theta + F_c (t) \cos \theta_s \\ M_s (\dot{w}' - q'u') &\approx -Q' S C_N + M_s g \cos \theta + F_c(t) \\ B\dot{q}' &= Q' S D \left[C_m + \frac{q'D}{2V'} C_{mq} \right] - l_m F_c(t) \end{aligned} \quad (25)$$

During ejection the aircraft velocity magnitude remains at V_0 . The aircraft is pulling up at a constant angular rate $\dot{\theta}$. It is assumed that the angle of attack, α_0 , of the X axis remains constant.

Velocities of the store relative to the X,Z coordinates are introduced

$$\begin{aligned} u &= V_0 \cos \alpha_0 + \dot{\theta} z_0 + Z_s \dot{\theta} + U_s \\ w &= V_0 \sin \alpha_0 - \dot{\theta} x_0 - X_s \dot{\theta} + W_s \end{aligned} \quad (26)$$

From kinematics one can obtain in addition

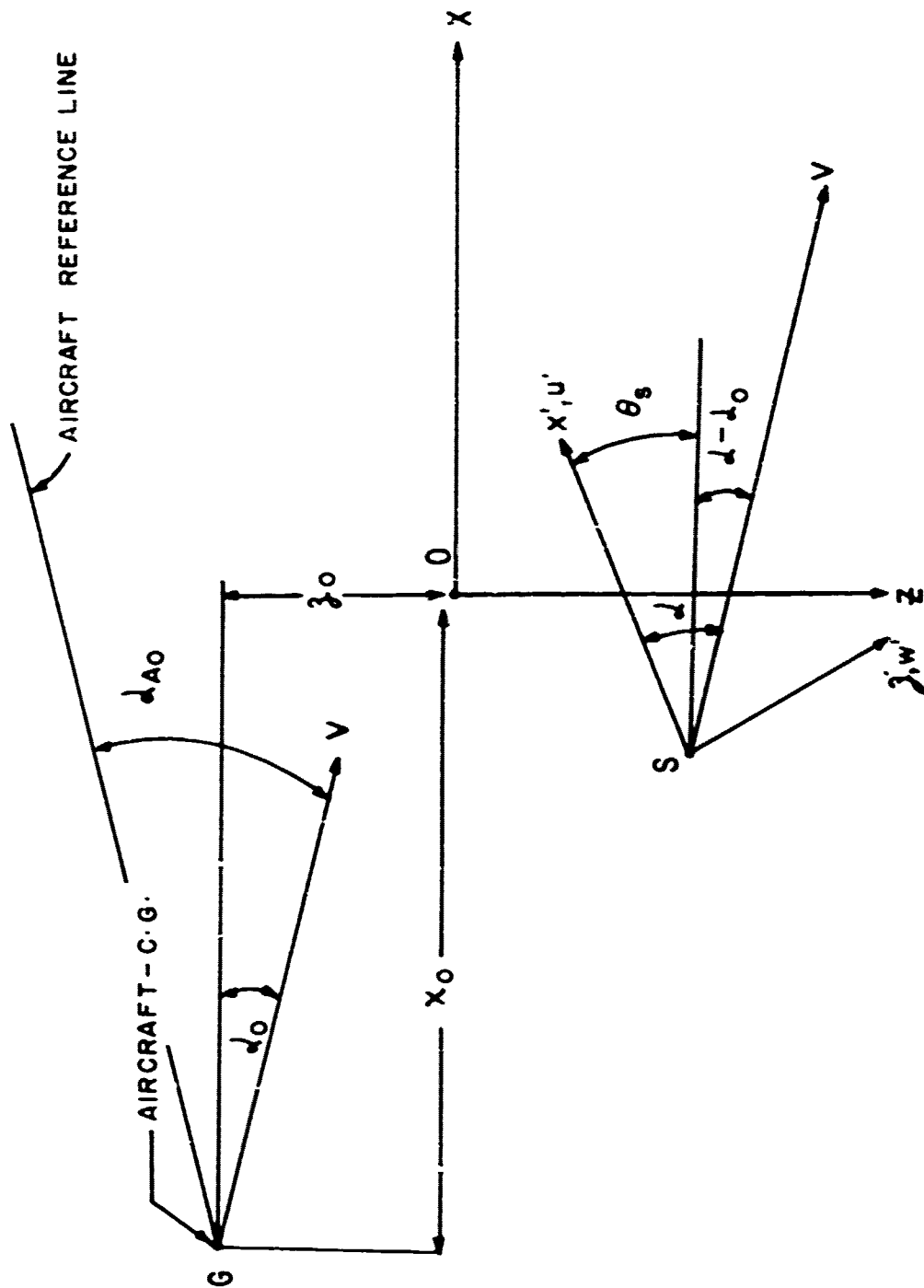


FIGURE 4

Ejection Phase Kinematic Geometry

$$\theta_1 = -\dot{\theta}t + \int_0^t q' dt + \theta_1(0)$$

$$\dot{\theta} = q' - \dot{\theta}$$

$$\ddot{\theta}_1 = \dot{q}'$$

$$\theta = \theta_0 + \dot{\theta}t + \theta_1$$

$$u' = u \cos \theta_1 - w \sin \theta_1$$

$$w' = w \cos \theta_1 + u \sin \theta_1$$

$$a'_1 = a_1 \cos \theta_1 - a_z \sin \theta_1$$

$$a'_z = a_z \cos \theta_1 + a_x \sin \theta_1 \quad (27)$$

It is assumed that the aerodynamic coefficients are given by a small perturbation from in-carnage coefficients.

$$C_A = -C_D = \text{const}$$

$$C_{mq} = \text{const}$$

$$C_N = C_{N_0} + (\alpha - \alpha_0) \frac{\partial C_N}{\partial \alpha} (\alpha_0) + \frac{\partial C_N}{\partial Z_1} Z_1$$

$$C_m = C_{m_0} + (\alpha - \alpha_0) \frac{\partial C_m}{\partial \alpha} (\alpha_0) + \frac{\partial C_m}{\partial Z_1} Z_1 \quad (28)$$

The partial derivatives are not readily measured or theoretically predicted. In applications C_{N_0} and C_{m_0} are free field values, the Z_1 derivative are zero. To good approximation.

$$\rho \approx \rho_0$$

$$V'^2 = u'^2 + w'^2 \approx V_0^2 \left[1 + 2 \cos \alpha_0 \frac{(\dot{\theta} z_0 + U_s)}{V_0} \right]$$

$$\approx V_0^2 [1 + O(<.01)] \approx V_0^2$$

$$Ma = \frac{V'}{C} \approx Ma_0$$

$$Q' \approx Q_0$$

$$\alpha - \alpha_0 \approx \frac{w'}{u'} - \alpha_0 \approx \theta_s + \frac{W_s - \dot{\theta} x_0}{V_0}$$

$$\sin \theta \approx \sin \theta_0 + \cos \theta_0 (\dot{\theta} t + \theta_s)$$

$$\cos \theta \approx \cos \theta_0 - \sin \theta_0 (\dot{\theta} t + \theta_s) \quad (29)$$

In addition

$$a_X = W_s \dot{\theta} + \dot{U}_s + \dot{\theta} w$$

$$a_Z = -U_s \dot{\theta} + \dot{W}_s - \dot{\theta} u \quad (30)$$

Equations (26)-(30) are applied to the set of equations (25) with retention of first order terms.

$$\dot{U}_s = -\dot{\theta}(2W_s + V_0 \alpha_0 - \dot{\theta} x_0) - \frac{Q_0 S C_{D0}}{M_s} - g \sin \theta_0 + \frac{F_c \phi}{M_s} - \frac{Q_0 S C_{N0} \theta_s}{M_s}$$

$$\dot{W}_s = \dot{\theta} V_0 + \frac{F_c}{M_s} - \frac{Q_0 S}{M_s} \left[C_{N0} + (\alpha - \alpha_0) \frac{\partial C_N}{\partial \alpha} + \frac{\partial C_N}{\partial Z_s} Z_s \right] + g \cos \theta_0$$

$$\ddot{\theta} = Q_0 S D \left[C_{m0} + (\alpha - \alpha_0) \frac{\partial C_m}{\partial \alpha} + \frac{\partial C_m}{\partial Z_s} Z_s + \frac{C_{ma} D}{2 V_0} (\dot{\theta}_s + \dot{\theta}) \right] - F_c [X_F - X_s + \phi(Z_s - Z_p)] \quad (31)$$

Solution of equation (31) begins with initial conditions (22). Computation ends when the vector has reached its end of stroke.

$$\ell_{pt} = \ell_e \quad (32)$$

The ejection velocity is defined as the store c.g. velocity vector at the end of stroke relative to its initial velocity vector. Components of the ejection velocity along and perpendicular to the initial velocity vector are

$$\begin{aligned} W_e &\approx W_s - \dot{\theta} V_0 t_e \\ U_e &\approx U_s \end{aligned} \quad (33)$$

W_e is normally what is called the ejection velocity since $U_e \ll V_0$.

IV. APPLIED COMPUTATIONS

The model developed in the sets of equations (1)-(33) was programmed for the CDC 6700 with general input variables. Specific applications are indicated below.

A. MER Structural Dynamic Data

Structural and geometric data were taken from References 7 - 11. The configuration chosen was a MER rack shifted completely forward. Parent rack was the MAU-9 (lugs and sway braces essentially in the same plane).

Eight collocation points are taken at x locations $x = 0., 1.08, 2.77, 4.65, 5.48, 7.60, 8.90,$ and 10.58 feet. Ejector unit connection points are at $x = 1.08, 2.77, 8.90,$ and 10.58 feet. Lug connections are at $x = 3.81$ and 6.31 feet. For collocation points on the strongback, I was computed as 43.1 in^4 ; for other points $I = 27.7 \text{ in}^4$. The elastic modulus for aluminum alloy 7075 is $E = 10.4 \times 10^6$ psi. Effective weights at collocation points (in corresponding order to x points above) were estimated as 5.43, 36.01, 36.79, 9.94, 9.94, 15.41, 38.28 and 34.13 lbs. Other pertinent geometric data are $l_c = .35, z_c = .479, z_q = -.259, z_1 = .252, z_2 = -.198, \Delta z = .2375, x_p = .6875, x_o = 5.,$ and $x_h = 1.6875$ ft. The remainder of the data are dependent on the configuration.

One check on the structural model is a comparison of a nose deflection computation with data from Reference 11. The configuration is 3 M-117 bombs forward and 2 shoulder M-117 bombs aft.

Test loads are taken from Reference 10. Loads on the forward stores are $F_{Z_c} = F_{Z_s} = 6.57 \times 10^3$ lbs., $F_{X_c} = F_{X_s} = -1.76 \times 10^3$ lbs., and $M_{Y_c} = M_{Y_s} = 5.31 \times 10^3$ in-lbs. Loads on aft stores are $F_{Z_s} = 6.54 \times 10^3$ lbs., $F_{X_s} = -1.76 \times 10^3$ lbs. and $M_{Y_s} = 7.61 \times 10^3$ in-lbs. Geometric data from References 7 and 12 are $x_c = .025, z_c = 1.25, z_s = .703,$ and $D = 1.333$ ft. Computation yields 2.8 inches for the nose deflection as compared to 3.1 inches from Reference 11. This indicates that the structural model is at least representative of the MER in pitch plane bending.

The ejector forcing function used is a mean curve for data for the Mk 2-1 cartridge taken from Reference 8. The forcing function compared with the original data is reproduced in Figure 5.

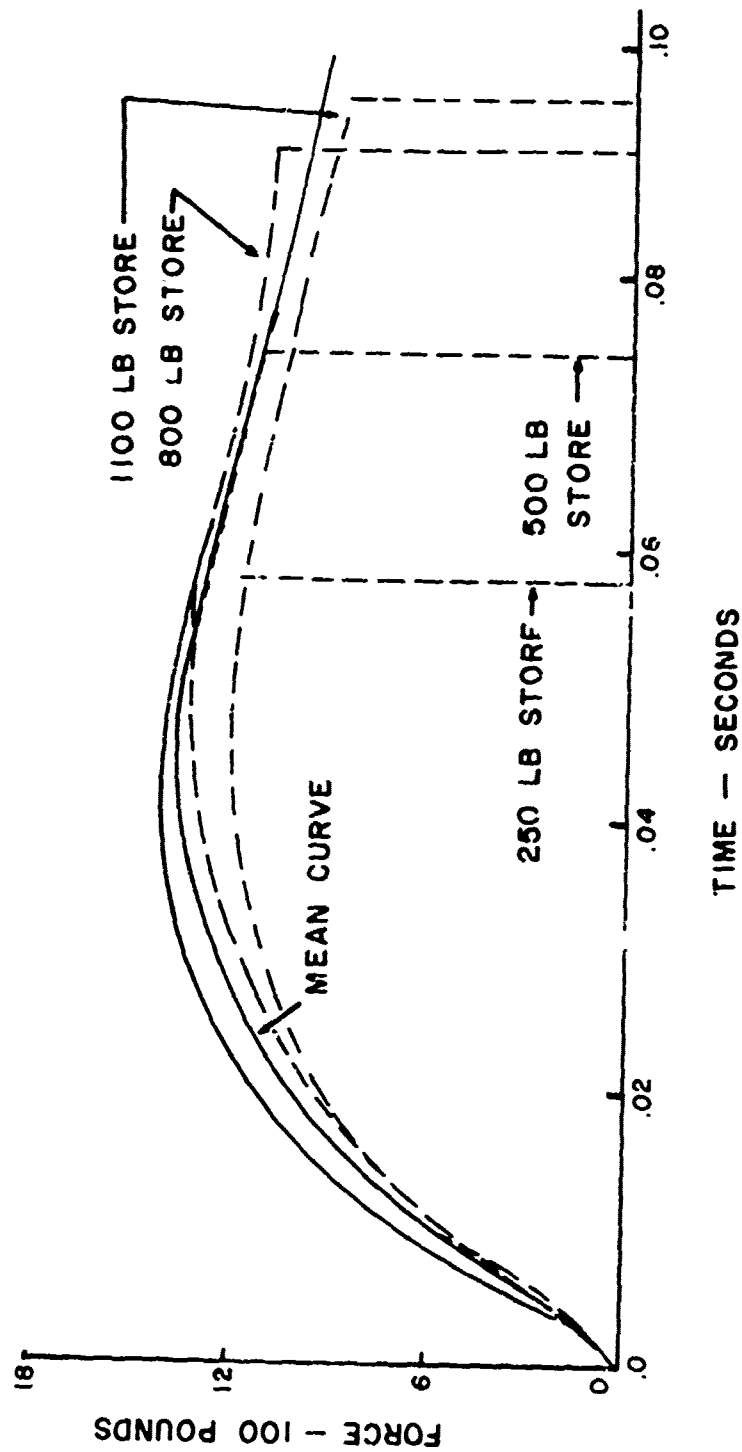


FIGURE 5

Force vs Time - MK 2 - 1 Cartridge

B. Applications

Sample computations were made corresponding to $\theta_0 = -30^\circ$, $Ma = .9$, and an altitude of 10000 ft. Corresponding air densities and speed of sound are based upon the ICAO standard atmosphere. The pull-up rate was varied between $\dot{\theta} = 0$ and $\dot{\theta} = .1$ rad/sec (about a 3.9 g normal acceleration).

As indicated previously, complete in-carriage aerodynamic data were available only for the M-117 (103 tail) bomb. Store inertial data from Reference 12 are $M_s = 824$ lbs. and $B = 1609$ lb-ft². Aircraft aerodynamic data were for an A7-D aircraft. The rack was located on the center wing pylon. Neighboring pylons were unloaded. Inboard and outboard shoulder in-carriage pitch plane aerodynamic loads were averaged to obtain equivalent pitch plane loads. The initial angle of attack of the aircraft reference line was assumed to vary linearly with $\dot{\theta}$ from 3.5° at $\dot{\theta} = 0$ to 10° at $\dot{\theta} = .1$ rad/sec. Angles of attack of the store are obtained by subtracting 3° (see for instance Reference 13).

Beam response was computed for a 6-bomb configuration and an aft centerline release with $\dot{\theta} = .1$. Because of the short distance from the rear lug to the ejector unit, deflections were small and induced velocities at the forward centerline store c.g. were small (.2 ft/sec maximum). Hence, flexibility for aft centerline releases was considered negligible. Induced ripple effects were also concluded to be negligible for this case. The remaining computations were made for a 5 bomb configuration and a forward centerline release.

Free field aerodynamics were obtained from References 12, 14 and 15. C_{N_α} and C_{m_α} were estimated from local slopes at the initial angles of attack.

Three modes were used to compute the MER beam response.

For the M-117 bomb computations were made for 3 cases

- (1) Flexible MER with variable $\dot{\theta}$.
- (2) Rigid MER with variable $\dot{\theta}$.
- (3) Rigid MER with $\dot{\theta} = 0$ but the same initial angles of attack as for (1) and (2).

Nominal ejection velocity for the M-117 from Reference 8 is about 6.4 ft/sec. First and second mode frequencies were computed as 44.3 and 68.9 rad/sec. In Figure 6 is plotted the flexible MER ejection velocity, W_e (Case (1)), and

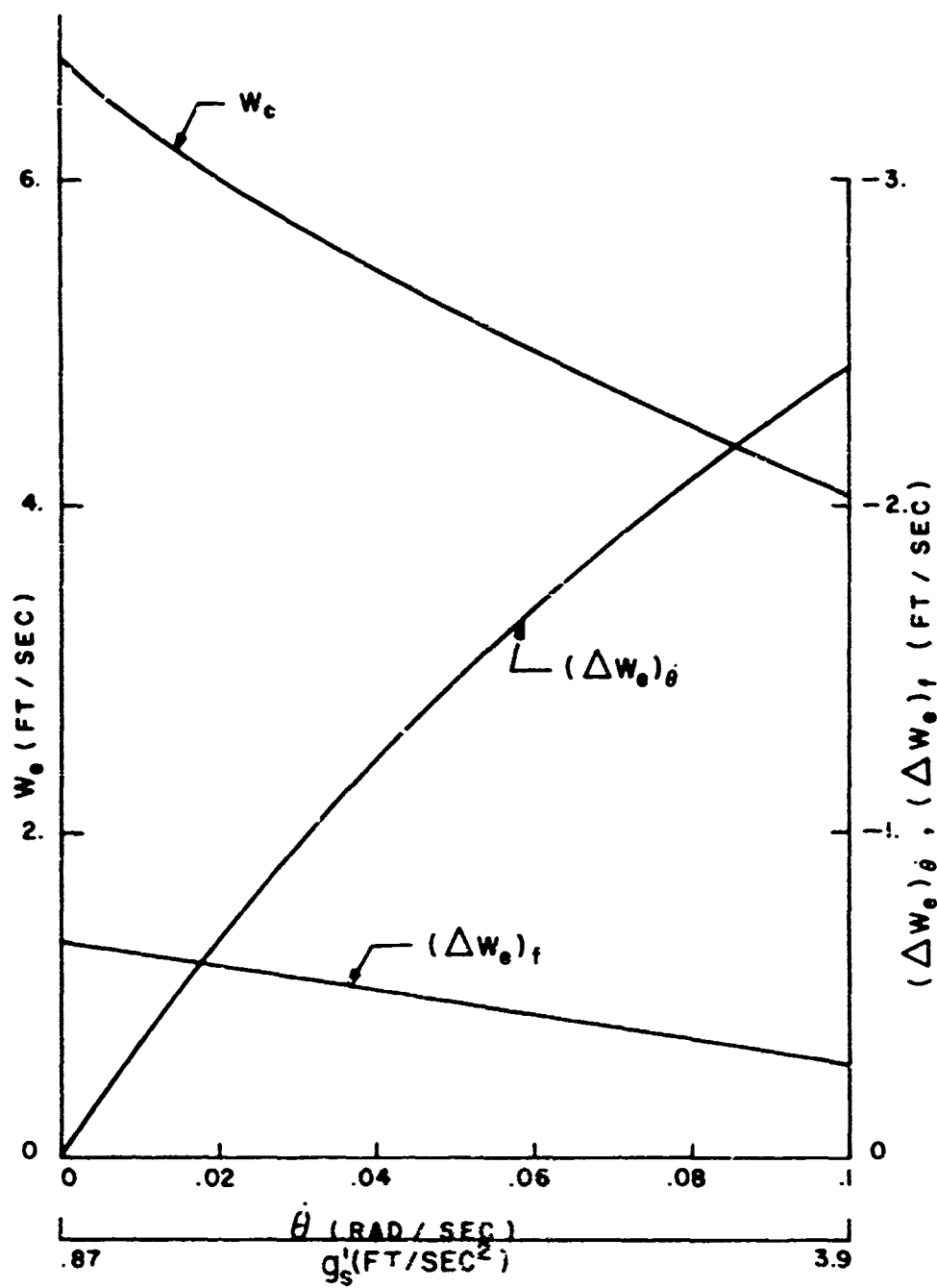


FIGURE 6

Ejection Velocity for M-117

contributions due to maneuver and flexibility. Contribution due to pull-up rate is labeled $(\Delta W_e)_u = W_{e2} - W_{e3}$. Contribution due to flexibility is labeled $(\Delta W_e)_f = W_{e1} - W_{e3}$. Figure 7 is a plot of angular rate, q_e (case (1)), $(\Delta q'_e)_u = q'_{e2} - q'_{e3}$ and $(\Delta q'_e)_f = q'_{e1} - q'_{e3}$. The numerical subscripts refer to the case numbers above.

For the M-117 flexibility decreases the ejection velocity due to aerodynamic, ejector, and gravity forces by some 8.7 percent at $\dot{\theta} = 0$ and lesser amounts for increasing pull-up rates. Note that the effect of pull-up rate alone is significant. For about .017 rad/sec maneuver and flexibility effects are about equal. Figure 7 shows that the ejection angular rate of the store is affected to a much greater extent by flexibility (30 percent for $\dot{\theta} = .1$). In this case, the angular rate magnitude is decreased or the effect of initial disturbances is alleviated.

Computations were also made on a hypothetical 300 lb. bomb in order to investigate effects of flexibility and maneuver on a more lightly loaded beam. The bomb was assumed to have the same aerodynamic coefficients and values of x_c as the M-117 bomb. Input variables which are different than the M-117 case are $B = 600 \text{ lb-ft}^2$, $D = .75 \text{ ft}$, $r_c = .958 \text{ ft}$, and $r_s = .497 \text{ ft}$. From Reference 8 the nominal ejection velocity is about 9.2 ft/sec. Frequencies of the first two modes are 71.9 and 113 rad/sec.

Figures (8) and (9) show that the maneuver contribution is still significant for the more lightly loaded MER beam.

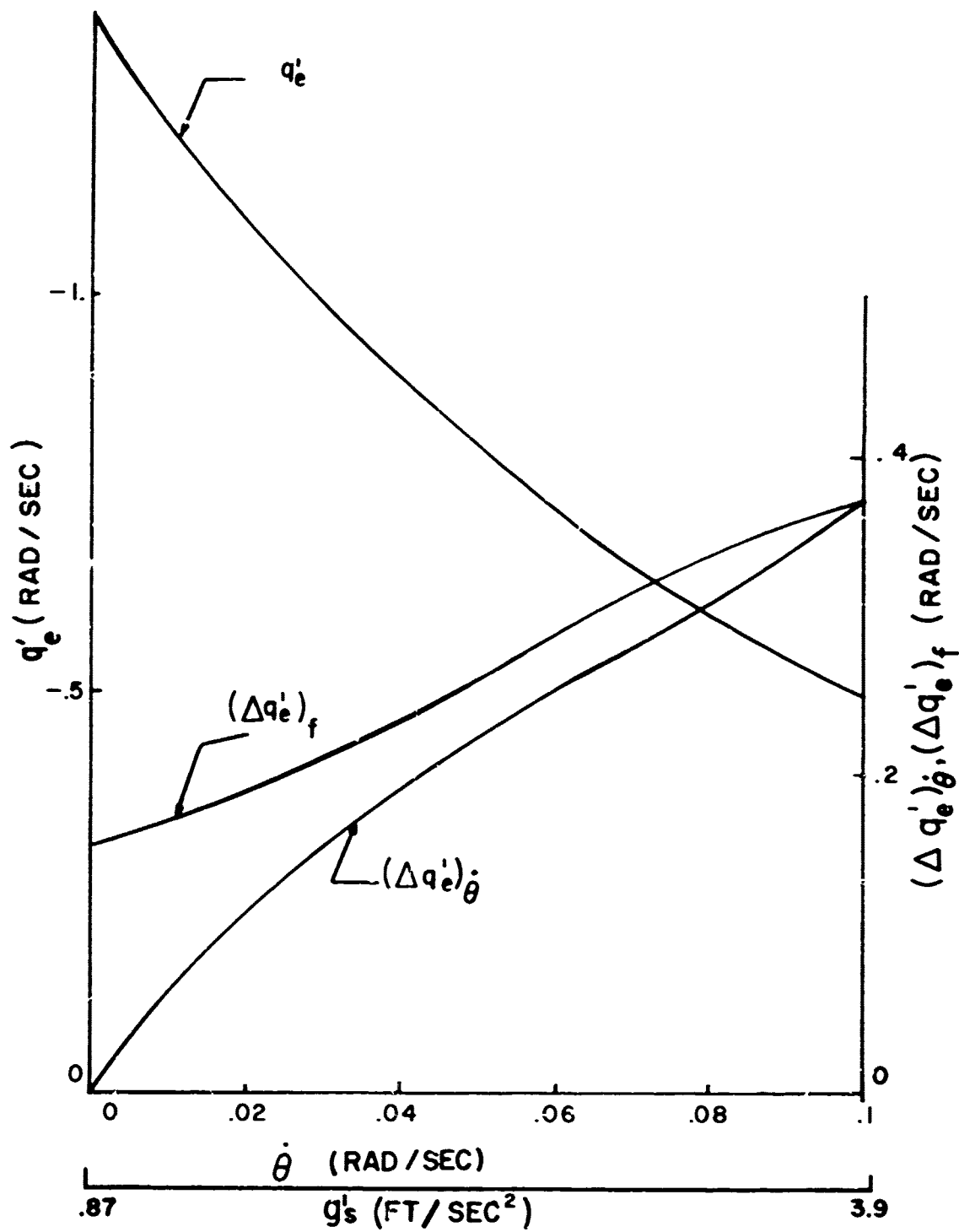


FIGURE 7

M-117 Ejection Angular Rate

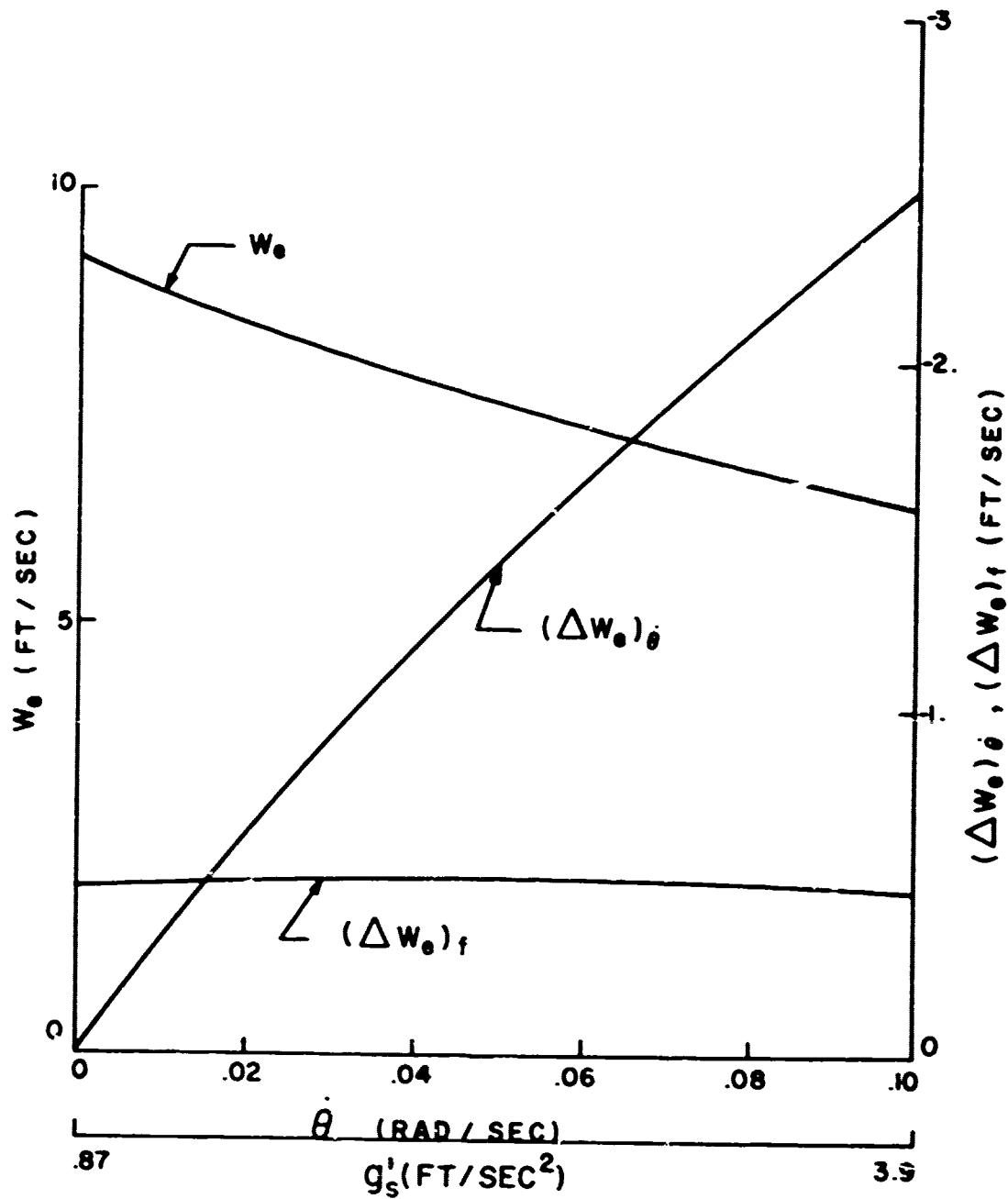
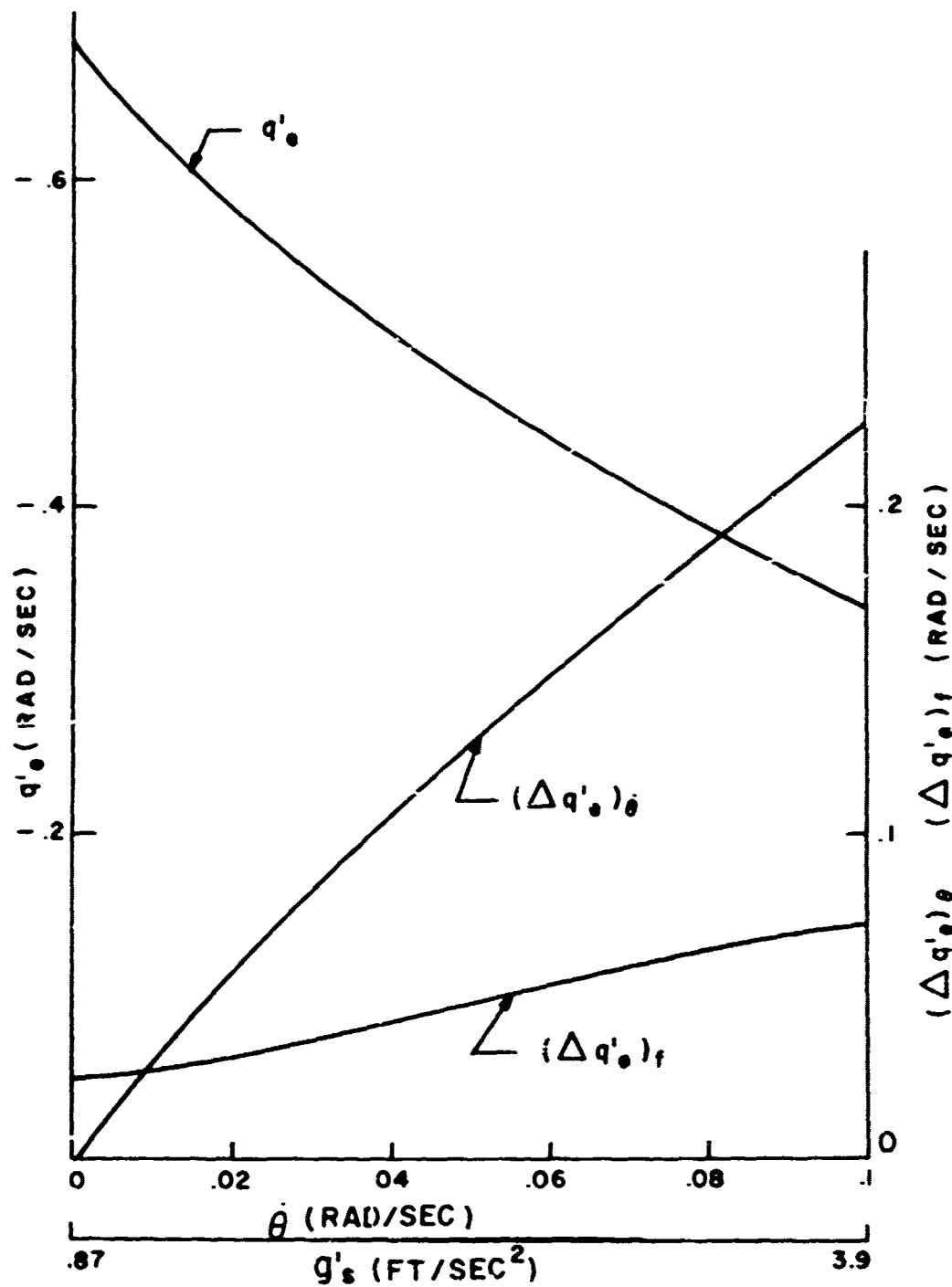


FIGURE 8

300 Lb. Store Ejection Velocity



300 Lb. Store Ejection Angular Rate

V. CONCLUDING REMARKS

A model has been developed which gives reasonable estimates of the effect of maneuver and MER flexibility on store ejection conditions for pitch plane motion. It appears that the ejection velocity is decreased by no more than 10 percent for the heaviest stores. However, the ejection angular rate due to interference aerodynamics, ejector force, and gravity is significantly affected.

In future work, it is hoped that results will be substantiated by experimental data on store kinematics and MER structural dynamics.

It is planned to extend this study to yaw plane and torsional deformation and six-degree-of-freedom motion. Yaw plane values of stiffness are about half that of the pitch plane. Parent rack sway bracing is relatively weak (especially for the AI-RO-7 rack). Hence, it is anticipated that the effect of MER deformation is more important for shoulder station releases.

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APPENDIX A
LIST OF SYMBOLS

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B	Pitch moment of inertia of store (slug-ft ²)
c	Speed of sound (ft/sec)
C_A	Axial force coefficient
C_D	Drag coefficient
C_{D0}	In-carriage drag coefficient
C_f	Force flexibility influence function (ft/lb)
C_{fij}	Force flexibility coefficient matrix (ft/lb)
C_m	Pitch moment coefficient
C_{m0}	In-carriage value of C_m
C_{mq}	Pitch damping coefficient
C_{Mik}	Moment flexibility coefficient matrix (lb ⁻¹)
C_N	Normal force coefficient
C_{N0}	In-carriage value of C_N
D	Store reference diameter (ft)
F_e	Ejection force (lb)
f_{xc}	x component of force reaction at hanger-beam connection point for a centerline MER station (lb)
F_{xc}	X component of force acting at centerline store c.g. (lb)
f_{xs}	Similar to f_{xc} for shoulder station (lb)
F_{xs}	Similar to F_{xc} for shoulder station (lb)
F_{x1}	F_{xc} for aft station (lb)

F_{X2}	F_{Xc} for forward station (lb)
F_{X3}	F_{Xs} for aft station (lb)
F_{X4}	F_{Xs} for forward station (lb)
F_z	Concentrated part of \bar{F}_z (lb)
\bar{F}_z	z component of total force acting at a MER beam cross section (lb)
F_{za}	F_z at an aft end of an ejector unit (lb)
F_{Zc}	Z component corresponding to F_{Xc} (lb)
f_{zca}	z component of force corresponding to f_{xc} for aft connection (lb)
f_{zcf}	Similar to f_{zca} for forward connection (lb)
F_{zj}	F_z at jth collocation node (lb)
\bar{F}_{zj}	\bar{F}_z at jth collocation node (lb)
F_{Zs}	Similar to F_{Zc} for shoulder station (lb)
f_{zsa}	Similar to f_{zca} for shoulder station (lb)
f_{zsf}	Similar to f_{zcf} for shoulder station (lb)
g	Acceleration of gravity (ft/sec ²)
I	Second moment of area of MER beam cross section (ft ⁴)
k	Sway brace spring constant (lb/ft)
K_{ci}	Flexibility matrix associated with ejector force (ft/lb)
K_{ij}	Flexibility matrix associated with beam acceleration (ft/lb)
ℓ	Beam length (ft); number of beam collocation points
ℓ_{pt}	Piston throw length (ft)

L_i	Deflection vector associated with time independent forces (ft)
Ma	Mach number
Ma_0	Initial Mach number
M_a	Couple at aft ejector support (ft-lb)
M_f	Couple at forward ejector support (ft-lb)
m_i	Effective mass at ith collocation node (slugs)
M_k	Couple at kth node (ft-lb)
M_{ea}	Similar to M_k for aft lug cross section (ft-lb)
M_{ef}	Similar to M_{ea} for forward lug cross section (ft-lb)
M_n	$= \sum_{i=1,2} m_i Z_{in} \dot{Z}_{in}$
M_s	Store mass (slugs)
M_{Yc}	Moment acting on centerline store (ft-lb)
M_{Ys}	Similar to M_{Yc} for shoulder store (ft-lb)
N	Number of modes in ξ_i expansion
p_z	Load distribution on MER beam (lb/ft)
q'	Angular rate of store (rad/sec)
Q	Dynamic pressure (lb/ft ²)
Q_0	Initial Q (lb/ft ²)
S	$= \frac{\pi D^2}{4} \text{ (ft}^2\text{)}$
t	Time (secs)
t_e	Ejection time (secs)

T_n	Time function associated with nth mode
u	X component of velocity of store (ft/sec)
u'	x' component of store velocity (ft/sec)
U_e	Component of ejection velocity parallel to initial aircraft velocity (ft/sec)
U_s	X component of velocity of store relative to X, Y, Z origin (ft/sec)
V'	Store velocity magnitude (ft/sec)
V_0	Aircraft velocity magnitude (ft/sec)
w	Z component of velocity of store (ft/sec)
w'	z' component of store velocity (ft/sec)
W_e	Similar to U_e for perpendicular component (ft/sec)
W_s	Similar to U_s for Z direction (ft/sec)
x	Longitudinal axis along MER (ft)
x'	Longitudinal store body axis (ft)
X	Axis parallel to x with origin at initial rigid c.g. position of store (ft)
x_a	Initial x distance from store c.g. to aft ejector unit support (ft)
x_f	Similar to x_a for forward support (ft)
x_c	Initial x distance from piston to rigid store c.g. (ft)
x_h	$= x_a + x_f$
x_p	x distance from aft support to ejector foot (ft)
X_p	X location of point on bottom surface of ejector unit adjacent to piston foot (ft)

X_s	X location of store c.g. (ft)
x_0	x distance from aircraft c.g. to in-carriage rigid c.g. of store (ft)
z	Axis perpendicular to x (ft)
z'	Axis perpendicular to x' (ft)
Z	Axis perpendicular to X (ft)
z_c	z distance from bottom of MER beam to initial store c.g. (ft)
z_e	z distance from bottom of MER beam to bottom of ejector unit (ft)
Z_{in}	nth mode vector
\hat{Z}_{in}	nth adjoint mode vector
z_l	z distance from neutral axis to lug (ft)
Z_p	Z component corresponding to X_p (ft)
z_0	z correspondent to x_0 (ft)
z_1	Distance from neutral axis to bottom of MER beam at ejector unit (ft)
z_2	Similar to z_1 , but distance to upper shoulder support (ft)
α	Store angle of attack (rads)
α_0	Initial value of α (rads)
Δz	z distance between shoulder supports (ft)
ξ_a	Beam deflection at aft support (ft)
ξ_f	Beam deflection at forward support (ft)
ξ_i	Deflection at ith collocation node (ft)
θ	Store elevation angle above horizontal (rad)

$\dot{\theta}$	Constant pull-up rate of aircraft (rad/sec)
θ_0	Initial elevation angle of store (rad)
θ_{\setminus}	Angle between x' and X axis
λ	Eigenvalue (sec/rad)
λ_n	nth eigenvalue (sec/rad)
ρ	Air density (slug/ft ³)
ρ_0	Initial ρ (slug/ft ³)
ϕ	$= \frac{\xi_a - \xi_f}{x_h}$ (rad)
ω_n	Frequency of nth mode (rad/sec)